Response to Report of Thomas Bryan

Moon Duchin
Professor of Mathematics, Tufts University
Collaborating Faculty in Race, Colonialism, and Diaspora Studies
Senior Fellow, Tisch College of Civic Life

December 20, 2021

1   Background and assignment

I am a Professor of Mathematics and a Senior Fellow in the Jonathan M. Tisch College of Civic Life at Tufts University. I have previously submitted an expert report in the current case and have been asked by counsel to provide additional material providing my opinion on the report of Thomas Bryan, particularly focused on his discussion of compactness metrics and of racial categories on the Census.

2   Compactness metrics

Part 4(D) of the Thomas Bryan report (pages 29-30) covers the topic of compactness metrics. In that Part, four compactness metrics are presented: Polsby-Popper, Schwartzberg, Reock, and Convex Hull.

2.1   Erroneous calculation

The Schwartzberg scores are calculated incorrectly in Mr. Bryan's report. Quoting the original 1966 paper where the score was proposed by Joseph Schwartzberg,[1]

> For any given two dimensional area the most compact shape is a circle. No other geometric figure has as low a ratio between its perimeter and area. The relative compactness of any other figure may be determined by finding the ratio of its perimeter to the perimeter of a circle of equal area. The ratio serves as an index of compactness. The index number of a circle is taken to be one. All other indices are higher and represent varying degrees of departure from perfect compactness. Thus, the index number of a perfect square is 1.13, of an equilateral triangle 1.29, and of a perfect five point star 1.95.

As this makes clear, the Schwartzberg score takes a minimum value of 1 (realized only for perfect circles); all other shapes have values above that. In the Thomas Bryan report, all districts are reported to have Schwartzberg scores less than one. Mr. Bryan supports his calculation by citing the website (`fisherzachary.github.io/public/r-output.html`) of an undergraduate student project, and including screenshots from that project in his report.

---

[1]Joseph E. Schwartzberg, Reapportionment, Gerrymanders, and the Notion of Compactness, 50 Minn. L. Rev. 443, 452 (1966).

2.2   Questionable combination

In addition to reporting scores incorrectly, Mr. Bryan also performs an operation that violates best practices in statistics and mathematical modeling: he adds scores that are in different units to create a "Total." Polsby-Popper scores are in dimensionless units that can be interpreted as a proportion of a certain circle's area; Reock scores are in proportion of a different circle's area; Convex Hull scores are in percentage of a certain polygon's area. It is unclear how one might interpret their sum, as the standard practice in quantitative analysis would be to only compute sums and averages of scores in like units.

   The practice of summarizing multiple compactness scores of numerous districts in a single number is not just abstractly discouraged, but has a concrete impact: it serves to hide the fact that different compactness scores reward or penalize different kinds of features. This can mislead readers into thinking that two plans are directly comparable when in fact one is stronger in some ways while the other is stronger in other ways. In a case like this, the appropriate conclusion would be that the compactness comparison is marked by tradeoffs.

3   Racial population categories

Part 3 of the Thomas Bryan report (pages 9-13) discusses Census Race Definitions, tallying population with categories that he calls "Black Alone" and "All Black." Mr. Bryan writes on p.10 that "the "alone" definition has been most defensible from a political science / Gingles 2 voting behavior perspective"—here, it is unclear what references support his claim, from political science or any other scholarly or practitioner literature.

   As Mr. Bryan acknowledges, the ability to use multiple categories to self-identify race in the Census is relatively recent. I note the Decennial Census treats Black as a checkbox, i.e., a yes/no question (see Figure 1). Thus, the Any-Part-Black definition (AKA "All Black") can be very simply described: *it contains all residents who, when presented with the Yes-or-No question about whether they are Black, answered Yes*.

Figure 1: The race question on the Decennial Census form in 2010.

I further note that Plan A, the first alternative plan presented in my report of December 10, has two majority-Black districts by any definition of Black that is plausibly used for VRA purposes: Any-Part-Black VAP, Black-Alone VAP, or Black Citizen VAP.[2]

	Black-Alone VAP	Any-Part-Black VAP	Black Citizen VAP
CD2	.5001	.5137	.5205
CD7	.5030	.5150	.5240

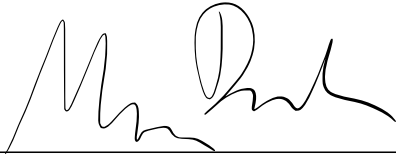Table 1: Statistics for CD2 and CD7 in Plan A

Future inquiry via voter registration

There is another source that could be useful to support the question of Black self-identification in Alabama: the voter registration file, in which citizens are asked to identify their race with a single choice. Counsel is currently attempting to secure a geocoded voter registration file. If I am provided with that resource in the near future, I hope to provide a supplemental report with the corresponding analysis.

---

[2]As explained in the supplemental material to my initial report, the BCVAP is estimated by using a special tabulation of the American Community Survey to calculate the citizenship rate for Black residents in the tract to which each block belongs, then applying that rate to the BVAP, in this case the Any-Part-Black VAP.

I declare under penalty of perjury that the foregoing is true and correct.

Executed this 20th day of December, 2021.

Moon Duchin